Forecast Error in the Canadian/U.S. Exchange Rate: Stronger Evidence of a Particular Type of Asymmetry

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Abstract

This study estimates a threshold autoregressive model of Canadian/U.S. exchange rate adjustment and finds that below the threshold, the process is approximately white noise. Above the threshold, errors are relatively more persistent, but they still dissipate quite rapidly. This is in contrast to the results of Samanta and Zadeh (2001) in which the forecast errors in the asymmetric model are more persistent than seems plausible given unit root tests of the errors. Additional data from recent years yields quantitatively similar estimates with even stronger evidence of asymmetry. An error correction model also gives more conclusive results concerning the adjustment process. The spot rate responds asymmetrically to the error while the forward rate responds mainly to lagged changes in the spot and forward rates.

I. Introduction

Research into questions of financial market efficiency has advanced in recent years with the development of more sophisticated time series models. Threshold autoregressive (TAR) models are being increasingly employed to analyze economic and financial data. They are especially useful in the study of time series which are characterized by asymmetric adjustment. In a seminal paper, Enders and Granger (1998), hereafter EG, develop a procedure for testing the null hypothesis of a unit root (symmetric adjustment) versus the alternative of stationarity with asymmetric adjustment. While their paper tested the spread between long and short term interest rates, the method can be applied to other interesting market efficiency questions such as purchasing power parity (PPP) or the unbiased forward rate hypothesis. Taylor (2001) makes a strong case for using TAR models to address the PPP question; a simple Dickey-Fuller test, which is known for its low power even when the model is correctly specified, has even lower power to reject the unit root null when the model is mis-specified.

A recent paper in this *Journal* (Samanta and Zadeh 2001) uses the EG procedure to test for asymmetric adjustment in the forecast error of the Canadian/U.S. exchange rate. While there is an established literature on the cointegration of spot and forward rates, the application of the EG procedure is novel and is of interest to researchers in international economics and finance. Furthermore, Samanta and Zadeh (2001) draw conclusions concerning arbitrage opportunities and exchange rate intervention policy from their study. These policy implications are clear enough to be understood by advanced undergraduates, which is another reason their paper is of interest. However, conclusions about policy implications in their study are subject to some of the same problems that have plagued the PPP literature.

Samanta and Zadeh (2001) begin by establishing that the forecast error (today's spot rate minus last period's forward rate) in the 1973-1994 data is stationary. Indeed, visual observation of the series suggests that the series is approximately white noise – a fact confirmed by the Dickey-Fuller test results (see Table 3 of Samanta and Zadeh (2001)). Using a different range of dates but the same variables, Zivot (2000) also finds that the forecast error is approximately white noise. However, the asymmetric model of Samanta and Zadeh (2001) appears to show more persistence. A reexamination of the data confirms that the forecast error is not very persistent. There is no unit root on either side of the threshold, and the asymmetry is quite significant statistically and economically when the threshold is chosen optimally. This finding is fundamentally the same, but much stronger, in a longer series of data from 1973 to 2003.

II. Model

Let s and f denote spot and forward rates, respectively, and let z represent the difference between this period's spot rate and last period's forward rate, $z_t = s_t - f_{t-1}$. It has been established elsewhere (e.g. Zivot (2000)) that the spot rate (s_t) and the lagged forward rate (f_{t-1}) are cointegrated in such a way that z is stationary. Starting from this basis, the approach is to fit a TAR model to the data and test for asymmetry against the null hypothesis of a symmetric AR(1) model.[1] The variable of interest is z, the forecast error. Thus, the basic model is

$$\Delta z_t = \rho z_{t-1} + u_t \tag{1}$$

The error term, u, is assumed to be normally distributed. The 2-regime TAR framework allows for a different speed of adjustment above and below a threshold, a.

$$\Delta z_t = I_t \rho_1 (z_{t-1} - a) + (1 - I_t) \rho_2 (z_{t-1} - a) + u_t \tag{2}$$

where

$$I_t = \begin{cases} 1 \text{ if } z_{t-1} \geq a \\ 0 \text{ if } z_{t-1} < a \end{cases} \tag{3}$$

After estimating (2) using an OLS regression, test the hypothesis $\rho_1 = \rho_2 = 0$. The F statistic under the null is not distributed in the usual way since this is essentially a test for a unit root. Compare the F statistic with the critical values found in EG. The test for symmetry ($\rho_1 = \rho_2$) can be done with a conventional F test.

Finally, the momentum-TAR or MTAR is estimated by using a different indicator function.

$$I_t = \begin{cases} 1 \text{ if } \Delta z_{t-1} \geq 0 \\ 0 \text{ if } \Delta z_{t-1} < 0 \end{cases} \tag{4}$$

The testing procedures are the same as for the TAR model.

For the TAR or MTAR, the modeler can choose the threshold. In some cases, thresholds of zero or the mean of the series are plausible. However, it is useful to adopt a procedure for choosing the optimal threshold. EG uses a result by Chan (1993) which says that choosing a to minimize the sum of squared errors in the regression yields a super-consistent estimator of the optimal threshold.

III. Data and Results

The data consists of monthly spot and forward rates obtained from Statistics Canada and stated in terms of U.S. cents per Canadian dollar.[2] I use two sample periods: 1973:10-1994:6 (as used by Samanta and Zadeh (2001)) and 1973:10-2003:12. Augmented Dickey-Fuller tests of the spot and forward series individually is unable to reject the unit root null in either series. Descriptive statistics for the variables are reported in Table 1a and 1b. Results for the two samples are reported in Tables 2 and 3, respectively.

Insert Table 1a and 1b about here

The Dickey-Fuller test reported in the first column of Table 2 specifies a model of symmetric adjustment and tests for a unit root. The unit root hypothesis is clearly rejected as the series of forecast errors is very nearly white noise. Each model was also estimated with lagged values of the dependent variable in the regression to take into account any serial correlation of the residuals, but in each case the AIC and the SC selected a model with no lags.

According to EG, the F test has greater power to reject the joint unit root null than the individual *t* tests. Therefore significance levels are indicated for the F tests of the asymmetric models. The F statistic for $\rho_1 = \rho_2 = 0$ must be compared to the critical values in EG. The null of joint unit roots is strongly rejected (1% level) in each case in both Tables 2 and 3. The estimated coefficients on ρ_1 and ρ_2 differ from each other similarly in each model with the widest gap appearing in the TAR model with $a = -0.19$.

The consensus of the four asymmetric models is that when the forecast error is negative, the process can overshoot back over the threshold $(\rho_2 < -1)$. This can actually be seen in the raw data of the series itself and was present whether sampling from the beginning or the middle of the month. This could be an interesting area for further study.

Insert Tables 2 and 3 about here

The TAR model with $a = -0.19$ fits the data the best in the sense of R^2, sum of squared residuals, and the Akaike criterion. The asymmetry is just barely significant at the 10% level for the TAR model and the MTAR with the optimal threshold. However, the TAR model with $a = -0.19$ produces a degree of asymmetry which is both statistically significant and quantitatively important. The fact that the optimal threshold is below the mean suggests that the overshooting phenomenon is particularly relevant for rather large negative deviations while anything above that (more than half of the observations) follows a somewhat more persistent process. Persistence is relative, of course, because even on the "persistent" side of the threshold about 70% of the deviation dissipates in one period in the absence of additional shocks.

Observations from the entire period reported in Table 3 confirm what is found in the earlier data. The difference between the coefficients in Tables 2 and 3 are not quantitatively important (mostly of order 10^{-2}). Also, with nearly a decade of additional observations, the ability to reject the null of symmetry is much improved under all specifications – a common feature of this class of models. In summary, the process is stationary on either side of the threshold (in contrast to the findings of Samanta and Zadeh (2001)), and the asymmetry appears to be quantitatively important and similar in the two sample periods.

Finally, I estimated a simple error correction model to gain additional insight into the dynamics of the adjustment. Various tests to determine the number of lagged variables to include in the model indicated that 2 to 7 lags was appropriate depending on the test. Results are shown with 5 lags. Samanta and Zadeh (2001) reported similar results, particularly regarding the response of the forward rate to lagged variables, but they did not draw any conclusions. However, the model does shed some light on how the variables respond to forecast error. The change in the forward rate is so dependent on the lagged changes that they appear to swamp any effect of the error correction term. The spot rate is the one that changes in response to the error, but most significantly (statistically and economically) when the error is negative. A spot rate that is 1 cent lower than predicted is associated with a 0.65 cent increase in the next period's spot rate. These changes in the spot rate feed into the lag structure and affect the forward rate in following periods. There is an interesting structure here that should prove useful for empirical and theoretical research.

IV. Conclusion

This analysis of exchange rate adjustment leads to several important observations. Each specification of the model confirms the high speed of the adjustment process which is apparent from a visual inspection of the forecast errors. The specific form of the asymmetry is also very clear. While errors above the threshold dissipate more slowly, errors below the threshold tend to produce overshooting as the rates adjust. The error correction model shows that it is the spot rate (not the forward rate) that appears to adjust more readily to these errors and does so in an asymmetric way. Also, it is interesting to note that the TAR and the MTAR give similar results (using the mean as the threshold).

This could be due to the forecast error having so little autocorrelation – the error and the change in the error often had the same sign. Changing the specification altered the indicator function in less than 1/3 of the observations.

Additional observations do not fundamentally alter the results. In fact, the main finding from the additional observations is that the null of symmetry can be more readily rejected. This is exactly what is expected given that the power of the test is sensitive to the number of observations.

Finally, in response to the conclusions of Samanta and Zadeh (2001), a few words about policy implications and the future of this research are in order. While these models can detect asymmetry, they do not reveal the precise mechanism by which it occurs. It would be erroneous to conclude that this means that central banks could pursue one type of policy (devaluation or revaluation) more readily than the other. The Lucas (1976) Critique applies here. We simply do not know enough about the reasons for what we are seeing. Nor is this by itself evidence of arbitrage opportunities. Answering that question would require information on other asset markets, most notably the 30 day bond market.

Future research should focus primarily on three areas: explaining the source of the asymmetry with theoretical models, exploring other TAR specifications provided that there is enough data to have sufficient testing power, and using higher frequency data with the TAR models to overcome both biases set forth in Taylor (2001). These inquiries could potentially be quite fruitful, and these methods can be used to examine a variety of asset markets.

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Endnotes

1. The reader is referred to Enders and Granger (1998) for the formal step-by-step details.

2. Statistics Canada provides daily data for these series. The daily series of forecast errors is the difference between period *t* spot rate and the period *t-30* forward rate. To test the robustness of the results, I used two different methods to convert the daily data to monthly data: first of the month and middle of the month. There was very little quantitative or qualitative difference between them. The paper reports results obtained by lagging the forward rate by 30 days and comparing the forecast error on the about the 15th of the month on a day for which there is an observation exactly 30 days prior.

	s_t	f_{t-1}	z
Mean	84.777	84.767	0.037
Median	83.900	83.900	0.044
Maximum	103.885	103.885	2.939
Minimum	71.291	70.877	-3.421
Std. Dev.	8.706	8.708	1.007
Skewness	0.602	0.594	-0.215
Kurtosis	2.578	2.560	3.189
Observations	249	249	249

Table 1a: Descriptive statistics for the 1973:10 to 1994:06 sample period

	s_t	f_{t-1}	z
Mean	79.875	79.858	0.040
Median	79.428	79.529	0.026
Maximum	103.885	103.885	4.633
Minimum	62.527	62.676	-3.421
Std. Dev.	10.455	10.458	1.039
Skewness	0.506	0.504	0.154
Kurtosis	2.591	2.578	3.766
Observations	363	363	363

Table 1b: Descriptive statistics for the 1973:10 to 2003:12 sample period.

	Dickey-Fuller (1)	TAR (2) and (3)	TAR (optimal threshold) (2) and (3)	MTAR (2) and (4)	MTAR (optimal threshold) (2) and (4)
ρ	-0.973				
	(-15.289)***				
ρ_1		-0.862	-0.697	-0.878	-0.858
		(-9.348)	(-6.319)	(-9.593)	(-9.251)
ρ_2		-1.074	-1.330	-1.061	-1.109
		(-12.281)	(-10.051)	(-12.033)	(-10.991)
a	0.037	0.037	-0.19	0.037	-0.17
R^2	0.486	0.492	0.506	0.490	0.493
SSR	251.287	248.473	242.229	249.197	248.422
AIC	2.859	2.856	2.834	2.859	2.860
SC	2.873	2.884	2.877	2.887	2.902
F $\rho_1 = \rho_2$		2.786*	9.308***	2.063	2.943*
F $\rho_1 = \rho_2 = 0$		119.109***	125.837***	118.406***	119.633***

Table 2: Results for the 1973:10-1994:06 sample; *t*-statistics are in parentheses. SSR=sum of squared residuals; AIC=Akaike info criterion; SC=Schwarz info criterion; MTAR and TAR models with the optimal threshold were estimated with a constant term in the regression. *** and * indicate significance at the 1% level and 10% level respectively for the F tests and the Dickey-Fuller *t*-test.

	Dickey-Fuller (1)	TAR (2) and (3)	TAR (optimal threshold) (2) and (3)	MTAR (2) and (4)	MTAR (0 lags) (optimal threshold) (2) and (4)	MTAR (1 lag) (optimal threshold) (2) and (4)
ρ	-0.940					
	(-17.877)***					
ρ_1		-0.828	-0.663	-0.799	-0.760	-0.668
		(-11.321)	(-7.520)	(-10.756)	(-9.813)	(-6.789)
ρ_2		-1.057	-1.335	-1.076	-1.138	-1.081
		(-14.127)	(-11.717)	(-14.710)	(-13.926)	(-12.050)
a	0.040	0.040	-0.16	0.040	-0.08	-0.08
R^2	0.470	0.477	0.491	0.480	0.484	0.487
SSR	388.902	383.793	373.378	381.391	378.690	376.271
AIC	2.915	2.907	2.883	2.901	2.897	2.896
SC	2.926	2.929	2.915	2.923	2.929	2.939
F $\rho_1 = \rho_2$		4.792**	15.083***	7.089***	9.822***	11.370***
F $\rho_1 = \rho_2 = 0$		163.870***	173.923***	166.035***	168.959***	85.173***

Table 3: Results for the 1973:10-2003:12 sample; *t*-statistics are in parentheses. SSR=sum of squared residuals; AIC=Akaike info criterion; SC=Schwarz info criterion; MTAR and TAR models with the optimal threshold were estimated with a constant term in the regression. *** and ** indicate significance at the 1% level and 5% level respectively for the F tests and the Dickey-Fuller *t*-test. The AIC selects a model with 1 lag under the MTAR (optimal threshold) specification.

	Δs_t	Δf_{t-1}
Δs_{t-1}	0.285	0.939
	(1.384)	(26.579)***
Δs_{t-2}	0.167	0.637
	(0.503)	(11.178)***
Δs_{t-3}	0.275	0.379
	(0.753)	(6.047)***
Δs_{t-4}	0.869	0.346
	(2.389)*	(5.546)***
Δs_{t-5}	0.572	0.227
	(1.840)	(4.252)***
Δf_{t-2}	-0.139	-0.650
	(-0.427)	(-11.618)***
Δf_{t-3}	-0.263	-0.375
	(-0.719)	(-5.989)***
Δf_{t-4}	-0.815	-0.346
	(-2.234)	(-5.539)***
Δf_{t-5}	-0.544	-0.218
	(-1.746)	(-4.086)***
Δf_{t-6}	0.017	-0.011
	(0.311)	(-1.202)
Constant	-0.230	-0.014
	(-2.282)	(-0.831)
$I_t(z_{t-1}+0.16)$	-0.032	0.051
	(-0.150)	(1.409)
$(1-I_t)(z_{t-1}+0.16)$	-0.649	0.061
	(-2.746)**	(1.516)
R-squared	0.054	0.973
Sum sq. resids	363.240	10.675
F-statistic	1.666	1032.855

Table 4: Error Correction Model for 1973:10-2003:12. *, **, and *** indicate statistical significance at the 1%, 5%, and 10% levels respectively (t-statistics in parentheses).