Value Added to Reservation Prices

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JEL: D42
Keywords: Market Structure and Pricing, Monopoly, Reservation Price

Abstract:

Suppose a price setting firm knows the distribution of reservation prices its customers have for an existing product. Then suppose the firm introduces a product improvement, and it is able to quantitatively evaluate the increase in performance (e.g. time saved, capacity increased, etc.) for the new product as compared to the original one. The paper provides a general method for pricing the innovation and then focuses on the case of normally distributed reservation prices. This approach can explain pricing behavior that standard linear demand curve models do not easily explain.

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*****This version is forthcoming in the Journal of Economics (MVEA)*****

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I. Introduction

Market demand curves are derived from some underlying information about potential customers. Often this information consists of a utility function which, when maximized subject to a budget constraint, yields a relationship between the relative prices of goods and the amount demanded. The present paper assumes that the underlying information is the distribution of reservation prices potential customers have for the product. For any given price, those customers whose reservation price is greater than or equal to the given price will purchase it. A price setting firm uses the information in the distribution of reservation prices to choose its optimal price. An important result is that the markup of price over marginal cost just equals the ratio of market penetration to the density of customers whose reservation price is the optimal price.

Our approach is especially useful in modeling situations in which the firm is able to make some change in the product (e.g. a technological improvement) that increases the reservation price that (at least some) customers are willing to pay for the product. There are potentially many ways to model the effects of such an improvement. For concreteness, we will focus on cases in which the improvement causes the parameters of the distribution of reservation prices to change in such a way that more could be sold at the original price. Such a change must surely improve profits, though it may induce the firm to raise or lower its price depending on how the parameters change.

As an example, consider Internet content provided via streaming audio/video. Suppose technological innovations improve sound and picture quality and thus increase the value of the content to the users with high reservation prices. Other users may not care as much about stereo sound and full-screen video. AM-quality audio meets their needs, and in fact, they would switch back to the radio if the price of the streaming audio/video is sufficiently high. The question for firms is how the characteristics of the distribution of reservation prices affect the price of the new and improved content. This paper will show that as long as the mean reservation price increases (while the variance does not decrease too much), the content providers will optimally increase prices as they target those willing to pay. The Internet example is particularly relevant to our value oriented approach because the marginal cost of disseminating information electronically is nearly zero before and after the quality change. Shapiro and Varian (1999) effectively make the case that value, not marginal cost, is the driver in information markets.

In contrast, consider products such as certain consumer electronic devices that also have a relatively low marginal cost. Improvements in the product might increase the average reservation price but concentrate the distribution of reservation prices, actually decreasing the variance. This would be the case, for example, if buyers with a higher reservation price are relatively less affected by the improvement, but buyers on the low end increase their reservation prices because of new applications or because the product is more user friendly. In this case, the profit maximizing strategy might well be to maintain or even lower the price as quality improves. The latter result is even more likely if the improvement in a product causes high end users to actually decrease their reservation price, perhaps because some functionality was sacrificed. Making a product easier to use can make it more difficult to "get under the hood" and modify it. Often, this is by design as in consumer electronics and computer related equipment. Thus, it is possible that an improvement might increase profit while inducing the firm to actually lower the price and increase market penetration.

While thinking of distributions of reservation prices as the essential foundation for market demand is perhaps uncommon, exogenous distributions of reservation prices can be found in some literatures (e.g., auctions in Lunander (2002), bundling in Pindyck and Rubinfeld (2001)). An early use of distributions of reservation prices is in Rob (1985). In that paper, the distribution is endogenously determined from a distribution of search costs. The search approach is useful in a world of competitive sellers and it implies nondegenerate distributions of reservation prices even when the goods are homogeneous (as in Rob). In our approach there is a single seller which precludes search, so the distribution of reservation prices is assumed to be influenced by other factors (preferences, income, differences in the intrinsic usefulness of the good for different consumers, etc.). In practice, when competition is imperfect and goods are differentiated, search as well as these other factors would all play a role.

In the pricing literature to which the present paper is most closely related Anderson, de Palma, and Thisse (1992, pp. 69-76) show that a consistent system of demand functions can be derived from distributions of reservation prices. In a paper that employs the Anderson, de Palma, and Thisse framework with a uniform distribution of reservation prices, Benrud (2002) considers a duopoly consisting of an established high quality producer facing competition from a new entrant producing a low quality good at a low price. His model is motivated in part by Christensen (1997), which discusses the difficulties in formulating the demand for innovations in the disk drive and mechanical excavator industries. Benrud's model implies a first order condition which is similar to ours, but given its focus on competitive issues, does not consider the case of a high quality producer increasing its quality even more.

Other approaches to quality are found in the literature. Quality is often simply an exogenous attribute of a product as it is in our paper. Indeed, Lutz (1997) assumes exogenous linear market demand curves which could be derived from the model in Section II of this paper (assuming a uniform distribution). In a similar vein, Hegji (1995) assumes the product has a "generic" component (priced competitively) and a "quality" component over which the firm has some price-setting power. Pichler (1997) has quality as an argument of the utility function (and thus should be seen as in the Dixit (1979) tradition) and considers the question of how the usual price elasticity of demand is affected by a quality elasticity. In a rather different vein, quality is sometimes also modeled analogously to location in a Hotelling-type model in, for example, Reitzes (1992) and Shaked and Sutton (1987).

Our framework also admits the possibility of "disruptive" change as described by Christensen (1997). Disruptive change requires firms to move rapidly, often without time for detailed demand analysis. However, the decision makers will typically have an engineer's sense of how the improvement can be quantified for different types of customers (e.g. the new product can complete a small job in half the time required for the old product to do the job but may not do as well on large jobs). We argue that quantifiable changes in the distribution of customer values are a useful way to think about changes in demand and produce a richer set of pricing results than are obtained with standard monopoly pricing theory.

Section II develops the model. Section III considers changes in demand using the parameterized model and derives some general results. Section IV considers a specific distribution of reservation prices, the normal distribution. The results are quantitatively very different from those derived using linear demand curves. Section V concludes.

II. The Model

Suppose that for any arbitrary price, P, customers whose reservation price for the product is greater than or equal to P will buy it. There is a continuum of buyers on $[a,b]$ (where b may be infinite) whose reservation prices are distributed according to an arbitrary cumulative distribution function (cdf), $F(v)$, with $f(v)$ representing the corresponding population density function (pdf). Assume any distribution of interest is a twice continuously differentiable function of the mean and standard deviation, (μ,σ).[1] Reflecting the dependence on parameters in the notation, the quantity demanded is:

$$Q(P) = N\int_{P}^{b(\mu,\sigma)} f(v;\mu,\sigma)dv \tag{1}$$

noting that the upper bound may also depend on the parameters. From (1), quantity demanded is the total potential number of customers, N, times the fraction of the market whose reservation price is greater than P, that is, $(1-F(v;\mu,\sigma)|_{v=P})$. A price-setting firm, with constant marginal cost c, chooses P to maximize $\Pi = (P-c)Q(P)$.

The first order condition is

$$\frac{d\Pi}{dP} = (P-c)\frac{dQ}{dP} + Q(P) = 0\,. \tag{2}$$

From (1), $dQ/dP = -Nf(P;\mu,\sigma)$ and $Q(P) = N(1-F(P;\mu,\sigma))$, so that (2) becomes

$$(P-c)=\frac{1-F(P;\mu,\sigma)}{f(P;\mu,\sigma)}\equiv G(P). \qquad (3)$$

Although omitted from equations (2) and (3), for some distributions and parameter sets an endpoint solution might be optimal. Our focus will be on interior solutions.

The profit-maximizing price is chosen so that the markup of price over marginal cost just equals the ratio of market penetration to the density of customers whose reservation price is the optimal price. The latter, the right-hand term of (3), is the inverse of the "hazard function," commonly defined in the literature as $f(v;\mu,\sigma)/(1-F(v;\mu,\sigma))$, evaluated at the optimal price. Multiplying the numerator and denominator by the number of potential customers, N, $G(P)$ can also be thought of as the ratio of sales to marginal sales, or the ratio of all customers to marginal customers.

To illustrate the first order condition, suppose reservation prices are distributed uniformly on the interval $[a,b]$ so that the pdf is $f(v)=\frac{1}{b-a}$ and

$$Q(P)=N(1-F(P))=N\frac{(b-P)}{(b-a)}.$$

(Recall that for the uniform $a\equiv\mu-\sqrt{3}\sigma$ and $b\equiv\mu+\sqrt{3}\sigma$.) The right-hand side of the first order condition (3) for the uniform is

$$G(P)=\frac{1-F(P)}{f(P)}=\frac{\left(\dfrac{b-P}{b-a}\right)}{\left(\dfrac{1}{b-a}\right)}=(b-P). \qquad (4)$$

Figure 1 illustrates the optimality condition. With price on the horizontal axis, the figure shows the margin $(P-c)$ as well as (4). Equilibrium price is $P=\frac{b+c}{2}$ (unless

$\frac{b+c}{2} < a$) and $Q = \frac{N}{2}\frac{b-c}{b-a}$. When $\frac{b+c}{2} < a$, price is set at the lower bound of the distribution of reservation prices, $P = a$.

In the case of the normal distribution the illustration of the first order condition is quite similar. For the normal, $G(P)$ is

$$G(P) = \frac{\int_P^\infty \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(v-\mu)^2}{2\sigma^2}} dv}{\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(P-\mu)^2}{2\sigma^2}}} \quad . \tag{5}$$

A convenient property of the normal is that $\frac{dG}{dP} < 0$ (when thinking of μ and σ as given.) See Appendix I for the proof. Figure 2 illustrates the first order condition for the normal distribution. It is similar to Figure 1, except that $G(P)$, the customer to marginal customer ratio, is strictly convex rather than linear. (The shift in $G(P)$ shown in the figure will be considered in the next section.)

The second order condition is (suppressing (μ, σ) in the notation)

$$\frac{d^2\Pi}{dP^2} = -Nf(P)\left(\frac{1-F(P)}{f(P)}\frac{f'(P)}{f(P)} + 2\right)$$

and noting from the definition in (3) that

$$G'(P) = -1 - \frac{(1-F(P))f'(P)}{(f(P))^2}$$

it follows that

$$\frac{d^2\Pi}{dP^2} = -Nf(P)(1-G'(P)).$$

Finally $\frac{d^2\Pi}{dP^2} \le 0$ if and only if $G'(P) \le 1$, which is hereby assumed. When $G'(P) < 0$, the second order condition always holds (as is the case for the uniform and normal distributions; see Figures 1 and 2). When $G'(P) \ge 0$, the second order condition holds if and only if $G(P)$ is flatter than the margin line at equilibrium (crossing it from above), recalling that the slope of the margin line is one. [2]

The effect of a change in marginal cost on the optimal price can be seen from the first and second order conditions. From (3)

$$\frac{dP}{dc} = \frac{1}{1 - G'(P)} > 0$$

because for any solution that satisfies the second order condition, $1 - G'(P) > 0$.

III. Changes in Demand: Theoretical Considerations

The model accommodates two kinds of changes in demand. First, suppose the total market size expands, (N increases) but the distribution of potential buyers' reservation prices is unchanged. Equation (3) implies that price is unchanged while equation (1) implies that quantity increases; clearly profits increase. The second kind of change in demand is when N is unchanged but the distribution of potential buyers' reservation prices changes in such a way that more could be sold at the original price. (This is the sense in which we mean that "value" has been added to reservation prices.) For example, suppose that with parameters (μ_0, σ_0) the optimal price (from (3)) is P_0 and the optimal quantity is $Q(P_0; \mu_0, \sigma_0) = N\int_{P_0}^{b(\mu_0, \sigma_0)} f(v; \mu_0, \sigma_0)dv$. Now consider another set of parameters (μ_1, σ_1) such that

$$Q(P_0;\mu_1,\sigma_1) > Q(P_0;\mu_0,\sigma_0)$$

or equivalently

$$1-F(P_0;\mu_1,\sigma_1) > 1-F(P_0;\mu_0,\sigma_0).$$

The "improved" product has the same marginal cost as the original but there may be some fixed costs of the improvement.

Greater market penetration at the old price implies higher profits. From equation (3) since the profit margin is an increasing function of price, a necessary and sufficient condition for $P_1 > P_0$ is that

$$G(P_0;\mu_0,\sigma_0) = \frac{1-F(P_0;\mu_0,\sigma_0)}{f(P_0;\mu_0,\sigma_0)} < \frac{1-F(P_0;\mu_1,\sigma_1)}{f(P_0;\mu_1,\sigma_1)} = G(P_0;\mu_1,\sigma_1). \qquad (6)$$

To see why (6) implies (and is implied by) $P_1 > P_0$ consider Figure 2. (In the figure, $G(P;\mu_0,\sigma_0)$ is the solid $G(P)$ and P_0 is the intersection of that curve with the margin line; $G(P;\mu_1,\sigma_1)$ is the dashed $G(P)$.) Since the margin line is upward sloping, in the cases when $G(P)$ is always downward sloping, $G(P;\mu_1,\sigma_1)$ must intersect the margin line at some price which is above P_0. This is true in general for continuously differentiable distribution functions regardless of whether or not $G(P;\mu_1,\sigma_1)$ falls below $G(P;\mu_0,\sigma_0)$ for prices lower or higher than P_0. The same result holds for the cases when $G(P)$ is upward sloping, as long as the equilibria satisfy the second order condition because at equilibrium $G(P)$ is always flatter than the margin line.

It can be shown (see Appendix II) that a version of the (strict) monotone likelihood ratio property gives a sufficient condition for (6). Monotone likelihood ratio property dominance at prices greater than or equal to P_0 is also sufficient. For the uniform and normal distributions, if the standard deviation is held constant, an increase in the mean is a necessary and sufficient condition for an increase in price. When the standard deviation changes (with either the uniform or normal distribution), an increase in the mean is neither a necessary nor sufficient condition for the price to increase. These results will be shown for the normal distribution in the next section; the demonstration of these results for the uniform is omitted for brevity.

IV. Pricing Innovation with the Normal Distribution

The previous section showed that increases in the value of a product (as reflected in the reservation price distribution) always create the potential for increased profits, but two questions remain. The first is whether the increase in profits is large enough to cover the fixed costs of product development, and the second is what price to charge since the previous section has demonstrated that it is not even necessarily the case that the optimal price increases. Assume reservation prices have a normal distribution with $\mu > 0$ and $\sigma \leq \frac{\mu}{2}$. These assumptions ensure that most of the mass (over 97%) will be on the positive reals.

The normal has several advantages over the uniform distribution. It allows for variation in the density of customers with a given reservation price. Potential customers in the upper tail of the distribution might be thought of as loyal customers, the lower tail as potential customers the firm is unlikely to reach, and the customers between the tails as those that are relatively sensitive to changes in price. The market demand curve derived from a normal distribution has the kind of "sideways s" shape which a Fortune 100 heavy equipment company faces for its products, according to a former head of pricing.[3] To visualize this demand curve, invert the cdf for the normal, and then reverse the axes. Another way to think of the normal distribution is to suppose that the distribution of reservation prices is the sum of the distributions of reservation prices for a number of features of the product (productivity, durability, resale value, warranty, etc.) which are each distributed uniformly.

A parameters diagram with mean on the vertical axis and standard deviation on the horizontal axis is especially useful to organize the analysis since solutions now can be derived numerically but not analytically. Appendix III gives the formal derivation of the profit level curves shown in Figure 3 and both the profit and price level curves in Figure 4. Consider the profit level curves shown in Figure 3. The higher profit level curve (i.e., the curve with points A and B) shows the combinations of standard deviation and mean that yield (at an optimum) a higher level of profit, and the lower profit level curve (i.e., the one with point E) shows parameters that yield lower profit. Holding the standard deviation constant, an increase in the mean reservation price causes profits to increase.

Table 1 gives some numerical examples elaborating on Figures 3 and 4. At point A, the mean is 118.52 and the standard deviation is 8.18. For these parameters, the first order condition (3) implies the optimal price is 115 and the quantity sold is 0.67 ($N \equiv 1$ throughout). Thus, at point A the firm optimally prices below the mean and realizes two thirds of its potential market. At point B, on the other hand, where the mean is again 118.52 but the standard deviation is 25.49, the firm optimally prices above the mean ($P = 125.80$) and sells to only 39% of the potential market. For both points A and B the profits are equal to 10.

As suggested by these examples, points on the upward sloping portion of the profit level curve are where the firm optimally prices below the mean and strives for higher market penetration. Points on the downward sloping portion of the profit level curve are where the firm optimally prices above the mean and sells to less than half of the potential market. The highest point on any given profit level curve is where the firm sells to exactly half its potential market and the price is equal to the mean. In terms of profit, any point on a given profit level curve is equivalent to any other, but how those profits are realized varies considerably. The essence of a profit level curve is that there are many combinations of mean and standard deviation that yield the same level of profits—there are many paths up the mountain.

As a final example on Figure 3, point E is chosen so that it has the same mean as points A and B, but it is also the highest point on its associated profit level curve. For the analogous example in the table, the price is equal to the mean (118.52) and, as just mentioned, sales are 0.5. For all parameter pairs with mean 118.52, the standard deviation of 14.78 is the one that yields the *least* profits. For a given mean, a firm is better served (in terms of profits) to have a low standard deviation, in which case the firm goes for volume, or a high standard deviation, in which case it goes for margin. To see this concretely, consider Figure 4. (The A-B profit level curve is the same as in Figure 3; the C-D profit level curve reflects a higher level of profits.) Point a_1 has the same mean as point A (and B) but a lower standard deviation so that profits increase. Point b_3, on the other hand, has the same mean but a larger standard deviation so that profits again increase.

Recall that the goal of the parameters diagram is to examine the implications of innovation on pricing as well as on profits. We now turn our attention to pricing and price level curves. Consider the firm whose initial demand parameters are those at point A, i.e., a low price, high quantity seller. Suppose an innovation to the product costs one unit, so that profits must increase by at least a unit to recover the fixed costs. If the innovation leaves the mean unchanged, then for a firm initially at point A, the innovation must reduce the standard deviation to 5.74 or less. When the standard deviation is exactly 5.74, the firm lowers its price from 115 to 114.35 and increases sales from 0.67 to 0.77. The firm optimally lowers its price and goes for market penetration.

However, as is clear from Figure 4, other combinations of mean and standard deviation would also generate the additional dollar's worth of profits. The point C is where the price remains at 115, but sales are 0.73. Thus points A and C are along the same price level curve. Point a_2 illustrates yet another possibility. In this case, the unit increase in the firm's profit is generated by leaving the standard deviation the same at 8.18 and increasing the mean to 120. Thus, a firm whose initial demand parameters are at point A has many potential strategies for increasing profits. The level curves indicate, first, whether the new demand parameters after the innovation are such that profits will recover the fixed costs of innovation, and second, whether the innovation causes the firm to raise or lower its price.

Taking B as the initial point, we have already seen that a profitable innovation might increase the standard deviation (point b_3). At point B the firm was already pricing for margin rather than sales, but such an innovation would cause the firm to raise its margin further (from 25.8 to 31.57) and actually lose sales (from 0.39 to 0.35). Point D is where the price is the same as at point B (hence, B and D are on the same price level curve) and quantity increases (from 0.39 to 0.43). Point b_2 is where the standard deviation is unchanged while the mean increases, causing both price and quantity to increase.

Returning to the examples described in the introduction, the Internet audio/video example could be compared to the move from B to b_2 or b_3 (or a combination of the two). At least on the cutting edge of the technology, market penetration is likely to be low, and the innovation may increase the mean reservation price more at the high end (increasing the variance). Price will certainly rise. Quantity may rise or fall depending on the precise nature of the change. This type of innovation targets the users with high reservation prices. For markets with higher penetration ratios, the firm begins at point A. One possible scenario for innovation is reminiscent of the consumer electronics example. The firm may look for innovations that target the low end consumer (perhaps at the expense of the high end consumer—lowering the standard deviation) while raising the mean reservation price. This can be represented by the move from A to a_1 or a_2. Price may rise or fall.

We investigate one additional aspect of the optimal pricing of an innovation, restricting our attention to cases when the improvement in the product increases the average reservation price but the standard deviation is unchanged. (These cases are illustrated by the shifts from A to a_2 and B to b_2 in Figure 4.) In general, such an increase in the mean will increase price, but by less than the increase in the mean. To see this, totally differentiate the first order condition (3), allowing the mean and price to vary while holding the standard deviation constant and recognizing that for the normal distribution, $\frac{\partial G}{\partial \mu} = -\frac{\partial G}{\partial P} > 0$.

$$0 < \frac{dP}{d\mu} = \frac{\frac{\partial G}{\partial \mu}}{1 - \frac{\partial G}{\partial P}} = \frac{\frac{\partial G}{\partial \mu}}{1 + \frac{\partial G}{\partial \mu}} < 1.$$

In this case it turns out that it is possible to develop a simple decision rule firms can use. Consider Figure 5. The market penetration ratio is on the horizontal axis (the proportion of potential buyers who actually purchase the product). The vertical axis shows the increase in price for a given increase in average reservation price. (The 45 degree line is also shown.) *Ceteris paribus*, the market penetration ratio is heavily dependent on the mean of the distribution relative to marginal cost. Figure 5 was generated by holding cost constant and varying the mean to generate market penetration ratios from approximately 0 to approximately 1. When the firm has a very high market penetration ratio (*Q* approaching *N*), the optimal increase in price is nearly 100 percent of the increase in average reservation price. This can be attributed to the fact that when the market penetration rate is already high, there is less potential to gain by increasing it since price would have to drop too much. Notice that the firm captures a large proportion of the increase in value, leaving very little on the table for the buyer. Using the penetration rate as a proxy for the fraction of the increase in the mean that is taken as price works well, especially for penetration rates above 50%. Applied to the parameters in Table 1, this pricing heuristic does quite well for the change from A to a_2, indicating a price of 115.99 (recall that the actual price at a_2 is 115.96). From B to b_2, the rule yields a price of 126.77 (the actual price at b_2 is 126.86). The heuristic is robust to changes in the values of the parameters.

As a final observation, if prices were distributed uniformly, any increase in mean reservation price (standard deviation unchanged) implies that the price increases by half of the increase in mean reservation price. That is, the pricing rule for the uniform distribution is described by the horizontal line at 0.5 in Figure 5. This is the familiar result that shifting the price intercept of a linear demand curve up by 1 unit causes the profit maximizing price to increase by ½ unit (assuming constant marginal cost). While the qualitative results of the uniform and normal distributions are similar, the quantitative results are not—unless the penetration ratio happens to be about 50 percent.

V. Conclusion

Some useful observations arise from this investigation of demand based on the underlying distribution of reservation prices. Consider an established firm that has a good sense of their customers' reservation prices for its existing product. Then suppose it introduces an innovation which creates a change in the demand for its product. In our model, this change could be embraced by customers at either the high end of the distribution (as in the Internet content example) or at the low end (as in the consumer electronics example). The firm's engineers who are responsible for the innovation might well have a useful way of quantitatively evaluating the change in performance (e.g. time saved, capacity increased, etc.) for different customers. Armed with this engineering data, the firm's marketing and pricing specialists can use the analysis of this paper to determine the value of the improvement and how to price it.

In the event that many customers experience the same absolute increase in their reservation price (increased mean, no change in variance), the firm could use Figure 5 to determine the price increase. Our model shows the stark quantitative difference between the normal and uniform distributions of reservation prices with regard to pricing innovation. In a world where time is short and the firm does not have the luxury of an exhaustive demand analysis, Figure 5 is an important first step in helping the firm evaluate the prospects for its innovation.

More generally, changes in the distribution of reservation prices can explain pricing behavior that standard linear demand curve models do not easily explain. This result is evident in Figures 3 and 4. When the distribution of reservation prices can be expressed with two parameters, then a parameters diagram can be used for comparative statics. On the one hand, the case of an innovation prompting both an increase in price and quantity follows from some very plausible changes in mean variance pairs. On the other hand, we can also treat a case such as consumer electronics where innovation is accompanied by falling relative prices. Focus on parameters sheds light on what sort of innovations would cause the firm to target the high end or low end of the market more intensively. In the fast moving, low marginal cost, high fixed cost technology sector, such insight is particularly valuable.

Our analysis also points to some interesting future research possibilities. If the old product and the innovation coexist in the market for a time, then price discrimination would be a possibility. If another firm offers a similar product, then an oligopoly model could be developed.

Endnotes

1. The problem could be formulated more generally assuming any parameters and the primary results shown in more generality. For ease of exposition we will use the mean and standard deviation since we confine our attention to the uniform and normal distributions.

2. For example the Weibull distribution has an upward sloping $G(P)$ for some parameter values; however, the second order condition is still satisfied because at equilibrium $G'(P) < 1$.

3. Private communication to author.

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Appendix I

The Slope of $G(P)$ for the Normal Distribution

Recall equation (5) from the text

$$G(P) = \frac{\int_P^\infty \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(v-\mu)^2}{2\sigma^2}} dv}{\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(P-\mu)^2}{2\sigma^2}}}. \tag{5}$$

To see that $\frac{dG}{dP} < 0$ (when thinking of μ and σ as given), differentiate (5)

$$G'(P) = -1 + G(P)\frac{(P-\mu)}{\sigma^2}$$

so that

$$G'(P) < 0 \quad \text{if and only if} \quad G(P) < \frac{\sigma^2}{P-\mu}.$$

Now consider a slight extension of a result from Feller (1950, p. 166)

$$\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(P-\mu)^2}{2\sigma^2}} \frac{\sigma^2}{P-\mu} = \frac{1}{\sigma\sqrt{2\pi}} \int_P^\infty e^{\frac{-(v-\mu)^2}{2\sigma^2}} \left(1 + \frac{\sigma^2}{(v-\mu)^2}\right) dv > \frac{1}{\sigma\sqrt{2\pi}} \int_P^\infty e^{\frac{-(v-\mu)^2}{2\sigma^2}} dv.$$

Dividing the left-most term and the right-most term by the pdf results in $G(P) < \frac{\sigma^2}{P-\mu}$,

and so $G'(P) < 0$.

Appendix II

Monotone Likelihood Ratio Property

Distribution $f(v;\mu_1,\sigma_1)$ will be said to dominate $f(v;\mu_0,\sigma_0)$ in the sense of the (strict) monotone likelihood ratio property $(f(v;\mu_1,\sigma_1)\succ_{MLRP} f(v;\mu_0,\sigma_0))$ if and only if

$$\frac{f(v_0;\mu_1,\sigma_1)}{f(v_0;\mu_0,\sigma_0)}<\frac{f(v;\mu_1,\sigma_1)}{f(v;\mu_0,\sigma_0)}.$$

for all $v\geq v_0$. Notice this implies

$$f(v;\mu_1,\sigma_1)>\frac{f(v_0;\mu_1,\sigma_1)}{f(v_0;\mu_0,\sigma_0)}f(v;\mu_0,\sigma_0)$$

for all $v\geq v_0$. From Das and Khwaja (2004, p. 17), we observe that if $f(v;\mu_1,\sigma_1)\succ_{MLRP} f(v;\mu_0,\sigma_0)$, then for a given P_0,

$$\frac{1-F(P_0;\mu_1,\sigma_1)}{f(P_0;\mu_1,\sigma_1)}=\frac{\int_{P_0}^{\infty}f(v;\mu_1,\sigma_1)dv}{f(P_0;\mu_1,\sigma_1)}$$

$$>\frac{\int_{P_0}^{\infty}\frac{f(P_0;\mu_1,\sigma_1)}{f(P_0;\mu_0,\sigma_0)}f(v;\mu_0,\sigma_0)dv}{f(P_0;\mu_1,\sigma_1)}=\frac{1-F(P_0;\mu_0,\sigma_0)}{f(P_0;\mu_0,\sigma_0)}.$$

The (strict) monotone likelihood ratio property shifts $G(P)$ upward in Figure 2. This is sufficient (though not necessary) to shift $G(P)$ upward at the equilibrium, P_0. In fact, $G(P)$ shifts up at the equilibrium if $f(v;\mu_1,\sigma_1)\succ_{MLRP} f(v;\mu_0,\sigma_0)$ only on the interval $[P_0,\infty)$ since we are only integrating over that interval.

Appendix III

The problem for the firm is to choose P to maximize

$$\Pi = (P-c)\int_P^\infty \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(v-\mu)^2}{2\sigma^2}} dv \,.$$

Not surprisingly, it will be convenient to change variables $z = \dfrac{P-\mu}{\sigma}$, so the problem for the firm is to choose z to maximize

$$\Pi = (\mu + \sigma z - c)\int_z^\infty \frac{1}{\sqrt{2\pi}} e^{\frac{-\gamma^2}{2}} d\gamma \,. \tag{A3.1}$$

The first order condition is

$$-(\mu + \sigma z - c)\frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}} + \sigma\int_z^\infty \frac{1}{\sqrt{2\pi}} e^{\frac{-\gamma^2}{2}} d\gamma = 0 \,, \tag{A3.2}$$

the solution of which is the optimal z.

To construct the parameters diagram consider first the combinations of (σ, μ) that yield the same optimal z score, or equivalently the proportion of potential sales that are realized. That is, fix the level of z at some arbitrary z_0, and find the combinations of (σ, μ) that satisfy (A3.2):

$$\mu = c - \left(z_0 - \frac{\int_z^\infty \frac{1}{\sqrt{2\pi}} e^{\frac{-\gamma^2}{2}} d\gamma}{\frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}} \right)\sigma \,. \tag{A3.3}$$

The points (σ, μ) that yield z_0 are a line with the marginal cost for the intercept. (Since these are straight lines, they have been omitted from the parameters diagrams.) The slope is $\sqrt{\pi/2} \approx 1.25$ when $z_0 = 0$, i.e., when half the potential market is reached. When

$z=-1$ the slope is about 4.8; when $z=1$ the slope is about -0.35; when $z \approx 0.75$ the slope is zero.

The profit and price level curves cannot be written in closed form, but they can be written parametrically, using z as the index. Considering the profit level curve first, fix Π_0 in (A3.1), substitute from (A3.2) or (A3.3) for $(\mu + \sigma z - c)$ and simplify so that σ is indexed by z:

$$\sigma(z) = \Pi_0 \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}} \left(\int_z^\infty \frac{1}{\sqrt{2\pi}} e^{\frac{-\gamma^2}{2}} d\gamma \right)^{-2}. \tag{A3.4}$$

Next, index μ by z:

$$\mu(z) = c - \left(z - \frac{\int_z^\infty \frac{1}{\sqrt{2\pi}} e^{\frac{-\gamma^2}{2}} d\gamma}{\frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}} \right) \sigma(z). \tag{A3.5}$$

In Figure 3, the profit level curves are drawn by parametrically plotting ((A3.4),(A3.5)). The slope of the profit level function is most easily found by taking the total differential of (A3.1), thinking of $z = z(\sigma, \mu)$, and denoting $q' = \frac{-1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}$ and $q = \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{\frac{-\gamma^2}{2}} d\gamma$

$$(d\mu + zd\sigma + \sigma(\frac{\partial z}{\partial \mu} d\mu + \frac{\partial z}{\partial \sigma} d\sigma))q + (\mu + z\sigma - c)q'(\frac{\partial z}{\partial \mu} d\mu + \frac{\partial z}{\partial \sigma} d\sigma) = 0.$$

Combining terms and using the first order condition

$$\frac{d\mu}{d\sigma} = -z \ .$$

Using this result on the slope as well as the information from the z level curves, the maximum point on any profit level curve lies along the $z_0 = 0$ curve, and the profit level curves are concave down as shown.

The price level curve can be constructed parametrically in a similar manner.

Use $P_0 = \mu + z\sigma$, and define

$$g(z) \equiv \frac{\int_z^\infty \frac{1}{\sqrt{2\pi}} e^{\frac{-\gamma^2}{2}} d\gamma}{\frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}} \tag{A.3.6}$$

and rewrite (A3.2)

$$P_0 - c = \sigma g(z). \tag{A.3.7}$$

From the first order condition (A3.2)

$$\sigma(z) = \frac{(P_0 - c)}{g(z)}$$

and

$$\mu(z) = P_0 - z\sigma(z).$$

For the slope, totally differentiate (A3.7)

$$\frac{d\mu}{d\sigma} = -z + \frac{g(z)}{g'(z)} \tag{A.3.8}$$

Clearly the price level curve can be positively or negatively sloped. We have chosen our examples on the negatively sloped portion of the curve. From (A.3.6) and (A.3.8) we see that the slope is zero when $z \approx -0.84$, positive for z values that are less than -0.84, and negative for z values greater than -0.84. A z value of -0.84 corresponds to market penetration of about 80%. Above this level, profitable innovations are strongly biased in favor of increasing the price since there are few additional sales to capture.

Parameters	Price and Quantity	Profit	Change in P and Q
Low P, High Q (A) $\mu = 118.52$ $\sigma = 8.18$	$P = 115.00$ $Q = 0.67$	$\Pi = 10.00$	
(a_1) $\mu = 118.52$ $\sigma = 5.74$	$P = 114.35$ $Q = 0.77$	$\Pi = 11.00$	Price decrease Quantity increase
(a_2) $\mu = 120.00$ $\sigma = 8.18$	$P = 115.96$ $Q = 0.69$	$\Pi = 11.00$	Price increase Quantity increase
High P, Low Q (B) $\mu = 118.52$ $\sigma = 25.49$	$P = 125.80$ $Q = 0.39$	$\Pi = 10.00$	
(b_1) $\mu = 121.62$ $\sigma = 22.37$	$P = 124.97$ $Q = 0.44$	$\Pi = 11.00$	Price decrease Quantity increase
(b_2) $\mu = 121.03$ $\sigma = 25.49$	$P = 126.86$ $Q = 0.41$	$\Pi = 11.00$	Price increase Quantity increase
(b_3) $\mu = 118.52$ $\sigma = 33.50$	$P = 131.57$ $Q = 0.35$	$\Pi = 11.00$	Price increase Quantity decrease
(C) $\mu = 119.19$ $\sigma = 6.72$	$P = 115.00$ $Q = 0.73$	$\Pi = 11.00$	
(D) $\mu = 121.40$ $\sigma = 23.73$	$P = 125.8$ $Q = 0.43$	$\Pi = 11.00$	
(E) $\mu = 118.52$ $\sigma = 14.78$	$P = 118.52$ $Q = 0.50$	$\Pi = 9.26$	

Table 1. Comparative statics for the normal distribution, $(c = 100)$.

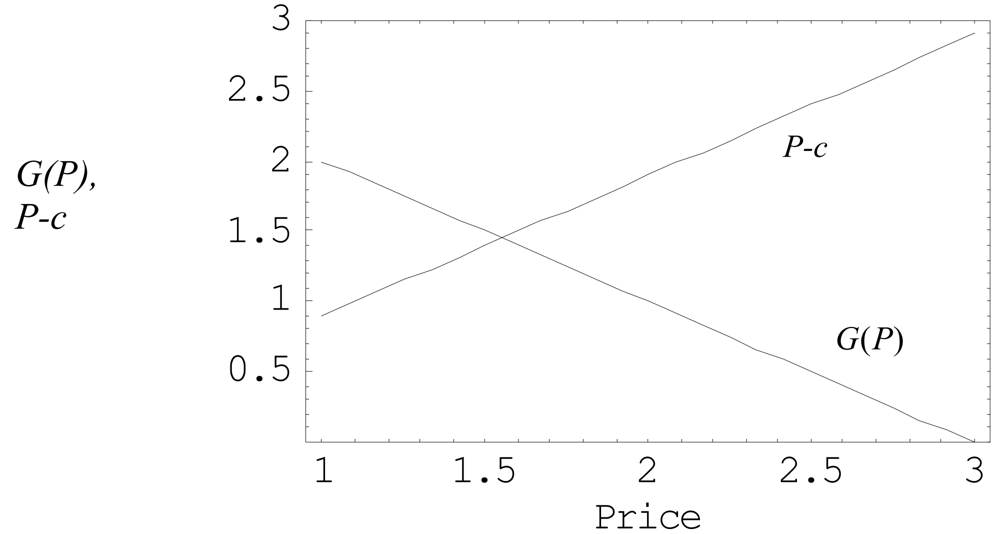

Figure 1. An equilibrium with a uniform distribution on [1,3] where $c = 0.1$.

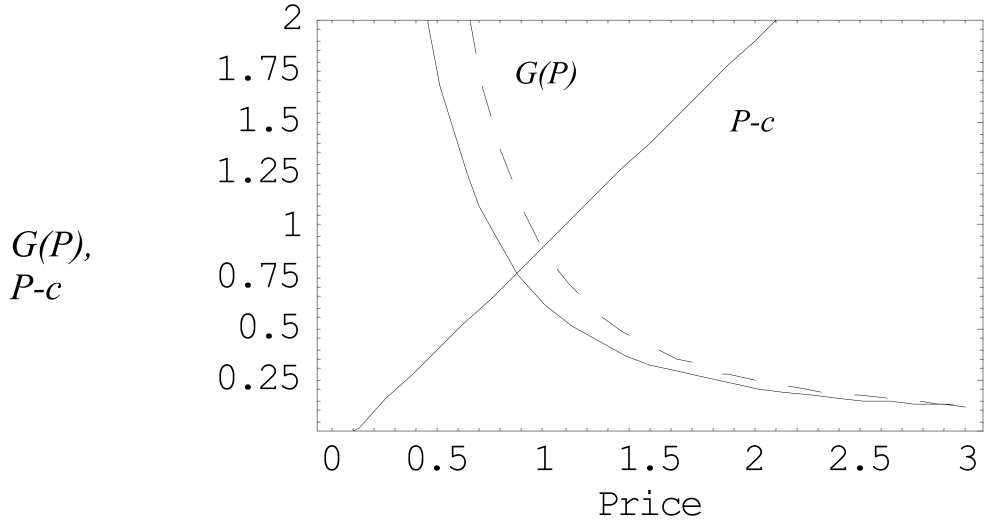

Figure 2. An equilibrium with a normal distribution having mean 1, standard deviation 0.5, and c = 0.1 (solid line). Dotted line illustrates an increase in the mean to 1.2 holding standard deviation and marginal cost constant.

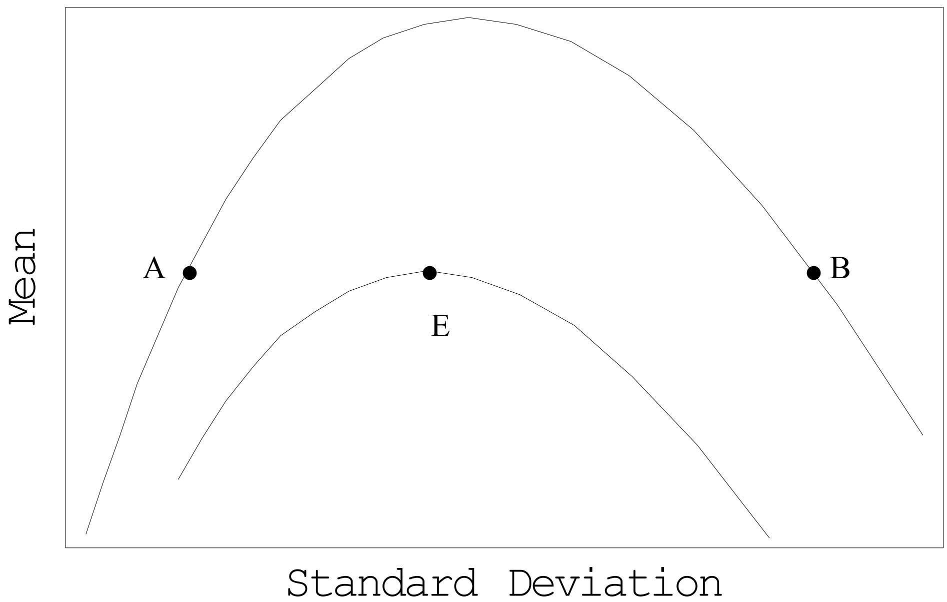

Figure 3. Profit level curves for a normal distribution.

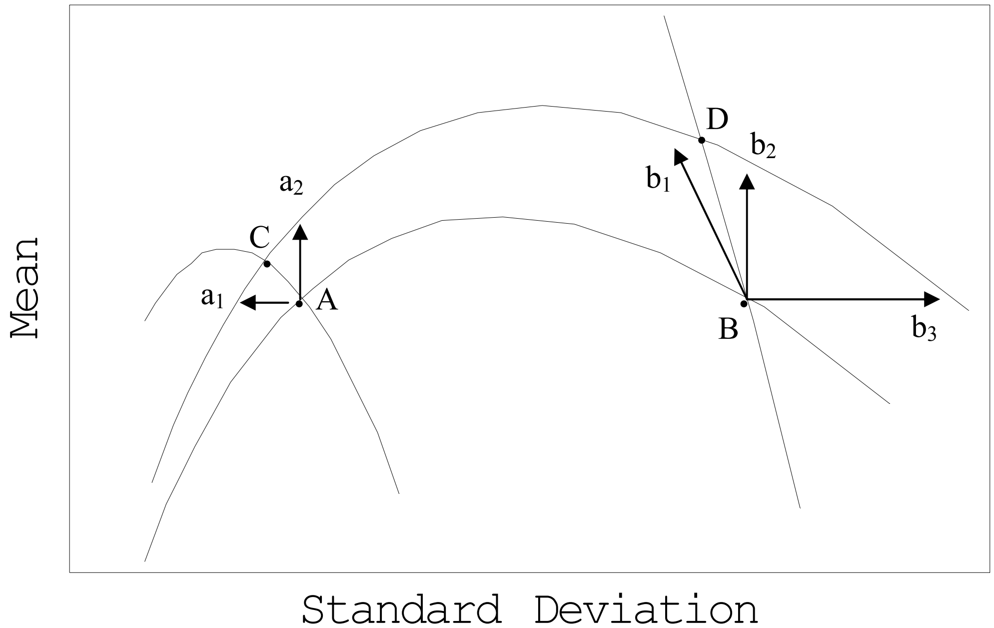

Figure 4. Graphic illustration of parameter shifts in Table 1.

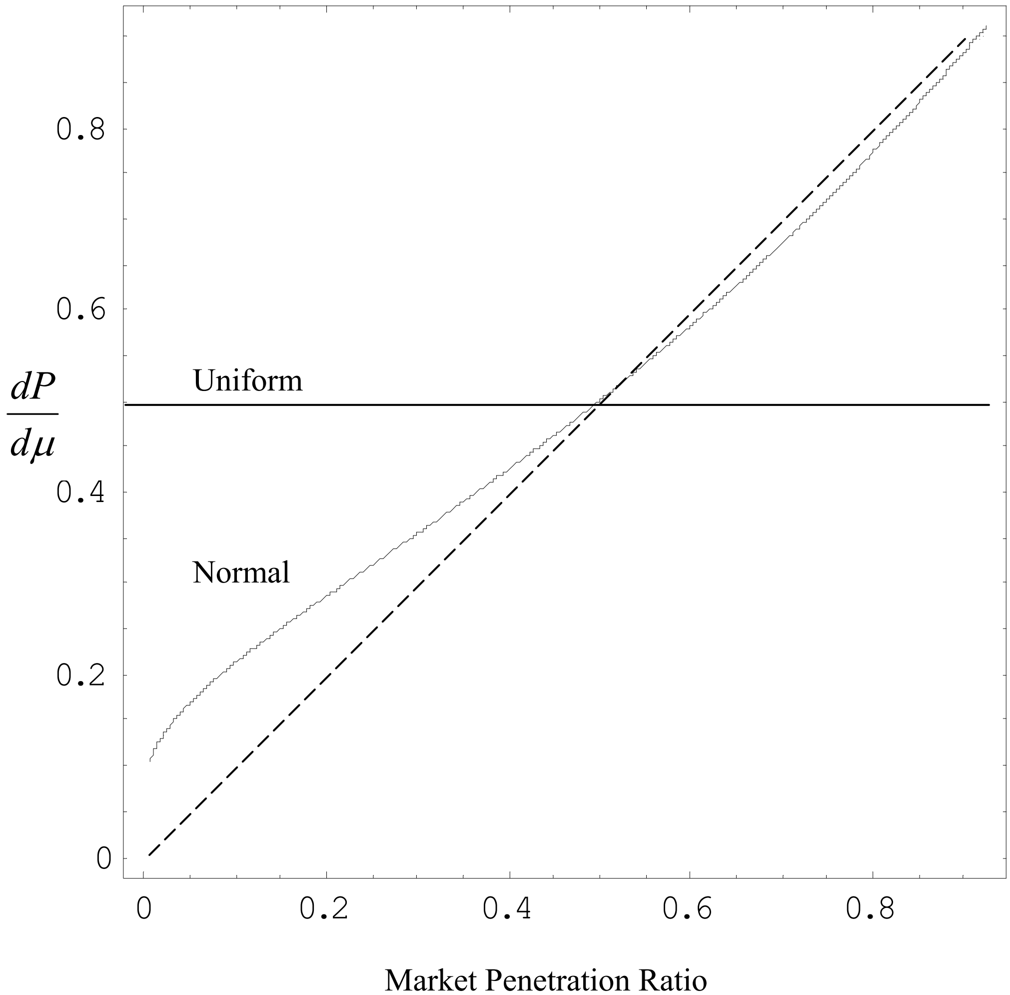

Figure 5. Pricing heuristic for the normal distribution. (Uniform shown for reference.)